Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–I • EXAMINATION – SUMMER • 2014

Subject Code: 610003 **Date: 18-06-2014**

Subject Name: Discrete Mathematics for Computer Science

Time: 10:30 am - 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Define the following terms with proper example: **[2+2+2+1]**
1. Universal Quantifier
2. Existential Quantifier
3. Tautology
4. Contradiction

(b) 1. Define Join Irreducible elements and Atoms with proper example. **04**
2. Define Meet Irreducible elements and Anti-atoms with proper example. **03**

Q.2 (a) **1.** Define Group. Check Whether <I, ×> forms a group or not where I is the set of Integers and × is multiplication operation. **03**
2. Define Sum-Of Product canonical form. Write Boolean Expression $x_1 * x_2$ in an equivalent sum-of-product canonical form in three variables x_1, x_2 and x_3. **04**

(b) Define left coset and right coset. Find the left cosets of {[0], [3]}in the group $<Z_6, +_6>$. State Lagrange's Theorem. **[2+3+2]**

OR

(b) Define equivalence relation. **07**
Let Z be the set of integers and R be the relation called Congruence modulo 5" defined by
$R = \{<x,y> \mid x \in Z \wedge y \in Z \wedge (x - y) \text{ is divisible by } 5\}$
Show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z.

Q.3 (a) Define Cyclic group. Prove that $<Z4, +_4>$ is isomorphic to $< Z_5, *_5>$ where $Z_5^* = Z_5-[0]$. **[1+6]**

(b) Find a minimal sum-of-product form using K-map **07**
(i) α (x, y, z) = xyz + xyz' + x'yz' + x'y'z
(ii) α (x, y, z) = xyz + xyz' + xy'z + x'yz + x'y'z

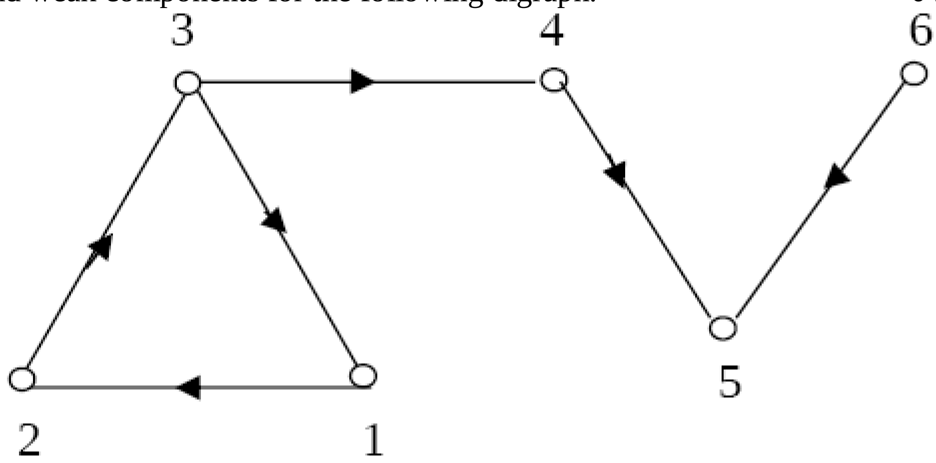
OR

Q.3 (a) Define Sub Boolean Algebra. Find all sub-algebras of Boolean algebra $<S_{30}$, *, ⊕, ',0,1>. Write proper steps. **07**

(b) (i) Show that in a lattice if $x \leq y \leq z$ then $x \oplus y = y * z$ **03**
(ii)In a lattice show that $(a * b) \oplus (c * d) \leq (a \oplus c) * (b \oplus d)$. **04**

Q.4 **(a)1.** Define weakly connected, unilaterally connected and strongly connected graphs. **03**

2. Define weak, unilateral and strong components. Find the strong, unilateral and weak components for the following digraph. **04**



(b) Draw di-graph and find in-degree and out-degree of each vertex from the given adjacency matrix. Using adjacency matrix, find total numbers of path of length 1 and 2 between each vertex. **07**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

OR

Q.4 **(a)** Define Sub-group. Find the subgroups of $\langle Z_{10}, +_{10} \rangle$ **07**

(b) Define “Lattice as an Algebraic System”, “Complete Lattice” and “Distributive Lattice. **[3+4]**

Let the sets S0, S1,………………, S7 be given by

S0 = {a, b, c, d, e, f}, S1 = {a, b, c, d, e}, S2 = {a, b, c, e, f}, S3 = {a, b, c, e}, S4 = {a, b, c}, S5 = {a, b}, S6 = {a, c},

S7 = {a}

Draw the diagram of $\langle L, \subseteq \rangle$,

where L = {S0, S1, S2,………, S7}

Q.5 **(a)** Describe the application of Boolean algebra to Relational Database with example. **07**

(b) Find minimal SOP using Quine Mc-cluskey method **07**

F(a,b,c,d) = Σ (0,1,2,5,6,7,8,9,10,14)

OR

Q.5 **(a)** **1**. Define Compatibility Relation, Partial order relation with proper example. **03**

2. Let R and S be two relations on a set of positive integers I, **02**

R = {<x,3x>/x ε I}

S = {<x,4x>/x ε I}

Then find R 0 R and R 0 S 0 R**.**

3. Let S = {2,4,5,10,15,20} and the relation ≤ is the divisibility relation. Draw the Hasse diagram of <S, ≤ >. **02**

(b) Show that the lattice <Sn,D> for n=100 is isomorphic to the direct product of lattice for n=4 and n=25. **07**

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