

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**PDDC - SEMESTER-I • EXAMINATION – WINTER 2013**

**Subject Code: X10001****Date: 17-12-2013****Subject Name: Mathematics - I****Time: 10.30 am - 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Each question carry equal marks (14 marks)

**Q.1** Do as directed.

(a) Using Gauss-Jordan method to find the inverse of a matrix  $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$  4

(b) Find the rank of matrix  $A$  by row echelone form, where 4

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$

(c) Find the orthogonal trajectories of the family of parabolas  $y^2 = 4ax$  4

(d) What is the order and degree of differential equation  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} / \frac{d^2y}{dx^2} = c$  2

**Q.2** (a) If  $u = \tan^{-1} \frac{x^3 + y^3}{x^2 + y^2}$ , prove that 5

i)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

ii)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u$

(b)  $4x - 2y + 6z = 8$  5

Solve  $x + y - 3z = -1$  by Gauss elimination method.

$$15x - 3y + 9z = 21$$

(c) Solve :  $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$  4

**Q.3**

(a) Find the eigen values eigen vectors of the matrix  $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$  5

(b) Find the maximum and minimum values of  $x^4 + y^4 - 2x^2 + 4xy - 2y^2$  5

(c) Solve:  $x \frac{dy}{dx} + y = x^3 y^6$  4

**Q.4** (a) Trace the curve  $y^2(a-x) = x^2(a+x)$  5

(b) The period of a simple pendulum is  $T = 2\pi\sqrt{l/g}$ , find the maximum error in  $T$  due to the possible error upto 1% in  $l$  and 2.5% in  $g$ . 5

- (c) If  $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$ , find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$  4
- Q.5** (a) Trace the curve  $r^2 = a^2 \cos 2\theta$ . 5
- (b) Evaluate  $\text{div} \bar{F}$  and  $\text{curl} \bar{F}$  at the point (1,2,3), where  $\bar{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$  5
- (c) Evaluate  $\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy$  by changing the order of integration. 4
- Q.6** (a) Find directional derivative of  $\phi = x^4 + y^4 + z^4$  at the point  $P(1, -2, 1)$  in the direction  $PQ$  where  $Q$  is  $(-1, 6, -1)$ . 4
- (b) Using Green's theorem, evaluate  $\int_C [(xy + y^2)dx + x^2 dy]$  where  $C$  is bounded by  $y = x$  and  $y = x^2$ . 4
- (c) Evaluate  $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) dx dy dz$  3
- (d) Evaluate  $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dy dx$  by changing to polar co-ordinates. 3
- Q.7** (a) If  $\bar{F} = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$ , evaluate  $\int_C \bar{F} \cdot d\bar{r}$  along the curve  $C$  in the  $XY$ - plane,  $y = x^3$  from the point (1,1) to (2,8). 4
- (b) Show that  $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$  where  $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ . 4
- (c) Find the value of 'a' if the vector  $\bar{F} = (ax^2y + yz)\bar{i} + (xy^2 - xz^2)\bar{j} + (2xy - 2x^2y^2)\bar{k}$  has zero divergence. 3
- (d) Show that the area between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$  is  $\frac{16}{3}a^2$ . 3

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