

Seat No.: \_\_\_\_\_

Enrolment No. \_\_\_\_\_

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**PDDC - SEMESTER-II • EXAMINATION – WINTER 2013**

**Subject Code: X20001****Date: 18-12-2013****Subject Name: Mathematics-II****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a)**      1) Define Beta function. Compute  $\beta(2.5,1.5)$ . 03

2) Prove that  $\int_0^1 x^3 (1 - \sqrt{x})^5 dx = 2\beta(8,6)$ . 04

**(b)**      1) Form the partial differential equation from  $z = ax + by + a^2 + b^2$  03

2) Solve  $\frac{\partial^2 z}{\partial x^2} = xy$  04

**Q.2 (a)**      Solve  $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$  07

**(b)**      Solve  $y'' + y = \tan x$  by the method of variation of parameter. 07

**OR**

**(b)**      Solve  $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log x)$ . 07

**Q.3 (a)**      Find the Fourier series expansion of  $f(x) = 2x - x^2$  in  $(0,3)$ ,  $f(x+3) = f(x)$ . 07

**(b)**      State Convolution theorem and using it, evaluate  $L^{-1}\left\{ \frac{s}{(s+2)(s^2+9)} \right\}$ . 07

**OR**

**Q.3 (a)**      Obtain a Fourier series to represent  $x - x^2$  from  $x = -\pi$  to  $x = \pi$ ,  $f(x+2\pi) = f(x)$ . 07

**(b)**      Using Laplace transform method, Solve  $y'' + y = t$ ,  $y(0) = 1$ ,  $y'(0) = -2$  07

**Q.4 (a)**      Express  $f(x) = x$  as a half range cosine series in  $0 < x < 2$ .  $f(x+4) = f(x)$ . 07

**(b)**      1) Find  $L\{(t+2)^2 e^t\}$ . 03

2) Find  $L^{-1}\left\{ \tan^{-1}\left(\frac{2}{s}\right) \right\}$  04

**OR**

**Q.4 (a)**      Find a Fourier series to represent  $x^2$  in the interval  $(-l, l)$ ,  $f(x+2l) = f(x)$ . 07

**(b)**      1) Find  $L^{-1}\left\{ \frac{1}{s^2 - 5s + 6} \right\}$ . 03

2) Find  $L\left\{ \frac{e^{-at} - e^{-bt}}{t} \right\}$ . 04

**Q.5 (a)**      1) Solve  $p + q = \sin x + \sin y$ . 03

2) Solve  $x(y-z)p + y(z-x)q = z(x-y)$ . 04

**(b)** Express the function  $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$  as a Fourier integral. Hence evaluate 07

$$\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$$

**OR**

**Q.5 (a)** Solve the equation  $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \quad u(x,0) = 4e^{-x}$  by the method of separation of variables. 07

- (b)**
- 1) Solve  $(D^4 - 4D^2 + 4)y = 0$  03
  - 2) Solve  $p(1+q) = qz$ . 04

\*\*\*\*\*