Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER–II • EXAMINATION – WINTER • 2014

Subject Code: X20001 **Date: 23-12-2014**

Subject Name: Mathematics - II

Time: 02:30 pm - 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Define Gamma and Beta Functions. Prove that $B(m,n)=\int_0^{\infty}\frac{x^{m-1}}{(1+x)^{m+n}}dx$. **05**

(2) Find $L(e^{-t}t^3)$. **02**

(b) Express $f(x)=x\cos x$ as a Fourier Series in $(-\pi,\pi)$. **07**

Q.2 **(a)** Using Laplace transforms solve the initial value problem $y''+y=\sin 2t$, where $y(0)=0 \& y'(0)=0.$ **07**

(b) Solve partial differential equation $pz-qz=z^2+(x+y)^2$. **07**

OR

(b) Solve: $x^2y''-4xy'+6y=21x^{-4}.$ **07**

Q.3 **(a)** (1) Find the Laplace transform of $t\sin t$ **03**

(2) Find the Laplace transform of $e^{-t}(t^2-2t+4)\sin t$ **04**

(b) Using the method of variation of parameters find the general solution of the differential equation $(D^2+2D+1)y=3x^{3/2}e^x$. **07**

OR

Q.3 **(a)** State the Convolution theorem for Laplace inverse transform . **07**

Using it find the Laplace Inverse transform of $\frac{s}{\left(s^2+a^2\right)^2}$.

(b) Solve the equation $u_x=2u_t+u$ given $u(x,0)=4e^{-3x}$ by method of separation of variables. **07**

Q.4 **(a)** Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \le x \le \pi$. Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - ..... = \frac{\pi^2}{12}$. **07**

(b) Using Beta-Gamma functions to prove that $\left(\int_0^{\infty} \sqrt{x} e^{-x^2} dx\right) \times \left(\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx\right) = \frac{\pi}{2\sqrt{2}}$. **07**

OR

Q.4 **(a)** Find the Fourier cosine transform of $f(x) = \begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, x > 2 \end{cases}$. **07**

(b) Solve $(D^2 + 2D + 10)y + 37\sin 2x = 0.$ **07**

Q.5 **(a)** (1) Eliminate the arbitrary function from the equation $z = xy + f(x^2 + y^2)$. **05**

(2) Solve : $\frac{\partial^2 z}{\partial x^2} = \sin x$ **02**

(b) (1) Define Z-transform. Find the Z-transform of the sequence $\{a^m\}, m \ge 0.$ **05**

(2) Prove that $L(1) = \frac{1}{s}$. **02**

OR

Q.5 **(a)** (1)Eliminate the arbitrary function from the equation **05**
$f(x + y + z, x^2 + y^2 + z^2) = 0.$

(2) Solve: $\frac{\partial^2 z}{\partial x \partial y} = x^3 + y^3.$ **02**

(b) (1) State the linearity property of Z-transform. Find the Z-transform of **05**
$f(k)$, where $f(k) = \begin{cases} 7^k, k < 0 \\ 5^k, k \ge 0. \end{cases}$

(2) State the relation between Beta-Gamma functions. **02**

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