

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC SEM-I Examination-Dec-2011

Subject code: X10001**Date: 19/12/2011****Subject Name: Mathematics-I****Time: 10.30 am -1.30 pm****Total marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) i. Determine the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. **02**

ii. If $x = r \cos \theta$, $y = r \sin \theta$ show that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$. **02**

iii. What is degree and order of $\frac{dy}{dx} = 2xy$? Solve it. **03**

(b) i. Discuss the nature of the origin for $y^2 = x(x+2) - 3$. **01**

ii. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} dy dx$ by changing into polar coordinates. **03**

iii. Find a vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. **03**

Q.2 (a) i. Solve: $x + 2y + 3z = 0$, $3x + 4y + 4z = 0$, $7x + 10y + 12z = 0$. **03**

ii. Using Gauss-Jordan method find the inverse of $B = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. **04**

(b) i. Find the eigen values of $C = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Find eigen vector corresponding to **04**

its smallest eigen value.

ii. If $u = e^{xyz}$, find u_{xyz} . **03**

OR

(b) i. If $D = \begin{bmatrix} 1 & 5 \\ 0 & -1 \end{bmatrix}$ find the eigen values of D^9 . **03**

ii. Find u_{xy} for $u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$. **04**

Q.3 (a) i. If $u = \sin^{-1} \frac{x^2 y^2}{x+y}$, show that $xu_x + yu_y = 3 \tan u$. **03**

ii. If $x = r \cos \theta$, $y = r \sin \theta$, prove that $JJ' = 1$. **04**

(b) Solve: **07**

i. $xy \frac{dy}{dx} = 1 + x + y + xy$

ii. $(x^2 - y^2) dx - xy dy = 0$

OR

Q.3 (a) i. Examine $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. **04**

ii. In estimating the cost of a pile of bricks measured as $2m \times 15m \times 1.2m$, the tape is stretched 1% beyond the standard length. Find the percentage error in the volume of the pile. **03**

(b) Solve : **07**

i. $(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^2$

ii. $(x^2 - ay) dx = (ax - y^2) dy$

Q.4 (a) If the stream lines of a flow around a corner are $xy = \text{constant}$, find their orthogonal trajectories. **04**

(b) Trace the curves **10**

i. $r^2 = a^2 \cos 2\theta$

ii. $x^2 y = a^2 (a - y) ; a > 0$

OR

Q.4 (a) When a resistance R ohms is connected in series with an inductance L henries with an e.m.f. of E volts, the current i amperes at time t is given by $L \frac{di}{dt} + Ri = E$. If $E = 10 \sin t$ volts and $i = 0$ when $t = 0$, find i as a function of t . **04**

(b) Trace the curves: **10**

i. $y^2 (2a - x) = x^3 ; a > 0$

ii. $r = a (1 + \cos \theta) ; a > 0$

Q.5 (a) i. Evaluate by changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy dx$. **03**

ii. By triple integration, find the volume of the sphere $x^2 + y^2 + z^2 = 1$. **04**

(b) i. Find the value of a if $(ax^2y + yz) i + (xy^2 - xz^2) j + (2xyz - 2x^2y^2) k$ is solenoidal. **03**

ii. Verify Green's theorem for $\int_C [(xy + y^2) dx + x^2 dy]$, where C is bounded by the curves $y = x$, $y = x^2$. **04**

OR

Q.5 (a) i. Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx$ **03**

ii. Find, by double integration, the area lying inside the curve $r = 2a \cos \theta$ **04**

(b) i. Find the curl of $(-2x^2y + yz) i + (xy^2 - xz^2) j + (2xyz - 2x^2y^2) k$. **03**

ii. Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2) dx + (x^2 + y^2) dy]$, where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = 1$. **04**
