Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC Sem-I June-July Examination 2011

Subject code: X10001 **Subject Name:** Mathematics - I

Date: 18/06/11 **Total Marks: 70** **Time: 10:30am to 1:30pm**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed.

(a) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point (-1, -1, 2). **(02)**

(b) Evaluate $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta$. **(03)**

(c) Trace the curve $y^2(a-x) = x^2(a+x)$. **(03)**

(d) Determine Rank of the following matrix by row echelon form **(03)**

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}.$$

(e) Using Gauss-Jordan method, find the inverse of following Matrix **(03)**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}.$$

Q.2 Attempt the following:

(a) Solve the following system of equations: **(03)**
x + y +2z =8;-x – 2y + 3z = 1; 3x -7y +4z =10.
By Gaussian elimination and back substitution.

(b) Find the Eigen values and Eigen vectors of the following Matrix **(04)**

$$A = \begin{bmatrix} 8 & 0 & 3 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

(c) Solve the following differential equations:

(i) $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ **(04)**

(ii) $(x^2 - y^2)dy = 2xydx$ **(03)**

OR

(c) Solve the following differential equations:

(i) $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$ **(04)**

(ii) $x\frac{dy}{dx} - ay = x + 1$ **(03)**

Q.3 Attempt the following:

(a) If u = $\log(x^3 + y^3 - x^2y - xy^2)$ prove that **(05)**

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}.$$

(b) If u = $\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{du}{dx} + y\frac{du}{dy} = \tan u$. **(05)**

(c) If $x = u(1-v), y = uv$ evaluate $\frac{\partial(x,y)}{\partial(u,v)}$. **(04)**

OR

Q-3 Attempt the following :

(a) If f(x,y,z)=$\log(x^2 + y^2 + z^2)$ **,**prove that $xfyz = yfzx = zfxy$. **(05)**

(b) Find the maximum and minimum values of the function $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$.

(c) Find the approximate value of $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$ at the point (3.01,4.02,11.98). **(05)**

(04)

Q.4 Attempt the following:

(a) Evaluate $\iint_R ydydx$, where R is the positive quadrant of the circle $x^2 + y^2 = 1$. **(05)**

(b) Find the volume of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **(05)**

(c) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx$. **(04)**

OR

Q.4 Attempt the following:

(a) Change the order of integration and evaluate **(05)**

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dydx.$$

(b) Change into polar co-ordinate and evaluate **(05)**

$$\int_0^a \int_y^a \frac{x^2 dxdy}{\sqrt{x^2 + y^2}}.$$

© By double integration ,find the area common to the Curves $y^2 = x$ and $x^2 = y$. **(04)**

Q.5 Attempt the following:

(a) A particle moves along the curve $x = 1 + t^3$, $y = t^2$ and $z = 2t + 5$. Find the components of its velocity and acceleration at time t=1 in the direction of $2\hat{i} + \hat{j} + 2\hat{k}$. **(05)**

(b) A vector field is given by $\vec{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ Show that $\vec{F}$ is solenoidal. **(05)**

(c) *show that the differential equation for the current i in an electrical circuit containing an inductance L and resistance R in series and acted on by an electromotive force* $E \sin \omega t$ *satisfies the equation* **(04)**

$$L\frac{di}{dt} + Ri = E\sin\omega t.$$

Find the value of the current at any time t,if initially there is no current in the circuit.

OR

Q.5 Attempt the following:

(a) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **(05)**

(b) Apply Green's theorem to evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$, Where c is the boundary of the area enclosed by the x axis and the upper half of the circle $x^2 + y^2 = a^2$. **(05)**

(c) Find the orthogonal trajectories of the family of parabolas $y = ax^2$. **(04)**

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