Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (OLD) EXAMINATION – SUMMER-2019

Subject Code: 110014 **Date: 06/06/2019**

Subject Name: Calculus

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) State Euler's theorem on homogeneous function. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin u\cos 3u$ **05**

(b) (i) If $u=xy^2+y^3+x^3+z^3$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=3u$ **03**

(ii) If $u=\ln\left(x^3+y^3+z^3-3xyz\right)$, prove that **03**

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$$

(c) Determine whether $\lim_{(x,y)\to(0,0)}\frac{x^2y}{x^4+y^2}$ exist or not? If they exist find the value of the limit. **03**

Q.2 (a) Find maxima and minima of the function **05**

$f(x,y)=x^3+y^3-3x-12y+20$

(b) Expand $e^x\tan^{-1}y$ about $(1,1)$ up to second degree in $(x-1)$ and $(y-1)$. **05**

(c) If $x=r\cos\theta, y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$ **04**

Q.3	**(a)**		Expand $\dfrac{e^x}{\cos x}$ in Maclaurin's series.	**05**
	(b)	**(i)**	Evaluate $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^2}$.	**02**
		(ii)	Find the values of a and b such that, $\lim\limits_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$	**03**
	(c)		Using Taylor's series find $\sqrt[3]{27.12}$ correct to four decimal places.	**04**
Q.4	**(a)**		Trace the curve $y^2(a+x)=x^2(3a-x)$	**05**
	(b)		Trace the curve $r=a(1+\cos\theta)$	**05**
	(c)		Using reduction formula evaluate $(i)\ \int_0^{\pi/2}\cos^5 dx$ and $(ii)\ \int_0^{\pi}\sin^6 dx$	**04**
Q.5	**(a)**	**(i)**	Test the convergence of $\sum_{n=1}^{\infty}\dfrac{1}{n^2+1}$	**02**
		(ii)	Test the convergence of $\sum_{n=1}^{\infty}\dfrac{n^2}{3^n}$	**03**
	(b)		Test the convergence of $\sum_{n=1}^{\infty} ne^{-n^2}$	**04**
	(c)		Obtain the reduction formula for $\int_0^{\frac{\pi}{2}}\cos^n x dx$	**05**
Q.6	**(a)**		Evaluate $\iint_R \sin\theta\, dA$, where R the region is in the 1st quadrant. i.e. outside the circle $r=2$ and inside the cardioids $r=2(1+\cos\theta)$.	**05**
	(b)		Evaluate by changing the order of integration $\int_0^1\int_{x^2}^{2-x} xy\, dydx$	**05**
	(c)		Find the volume bounded by cylinder $x^2+y^2=4$ and the planes $y+z=4$, $z=0$.	**04**

Q.7 **(a)** Prove that $\int_{1}^{\infty} \frac{1}{x^p} dx$, converges when $p > 1$ and diverges when $p \le 1$ **05**

(b) Use triple integral in cylindrical co-ordinate to find the volume of solid, bounded above the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by $xy - plane$ and laterally by the cylinder $x^2 + y^2 = 9$ **05**

(c) Find the volume of a cone with height $4cm$ and radius of base $4cm$. Use the method of slicing. **04**

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