Seat No.:________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 (NEW) EXAMINATION – WINTER 2017

Subject Code: 2110014 **Date: 03/01/2018**

Subject Name: Calculus

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Mark
Q.1		**Objective Question (MCQ)**	
	(a)	Choose the most appropriate answer out of the following options given for each part of the question:	**07**
	1.	The value of the $\lim_{x\to\pi} \frac{\sin x}{\pi - x}$ is ______ (A) 0 (B) 1 (C) π (D) -1	
	2.	The sequence $\left\{\left(\frac{1}{2}\right)^n\right\}_{n=1}^{\infty}$ converges to ____ (A) 0 (B) 1 (C) $1/2$ (D) 2	
	3.	The sum of the series $2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\ldots$ is ____ (A) 3 (B) 0 (C) $1/3$ (D) $2/3$	
	4.	For the curve $y^2(a-x)=x^3, a>0$, the origin is _____ (A) a Node (B) an isolated point (C) a Cusp (D) None of these	
	5.	If $u = y^2 f\left(x/y\right)$, then $x^2u_{xx} + 2xy\,u_{xy} + y^2u_{yy} =$ ____ (A) 6u (B) 0 (C) u (D) 2u	
	6.	If $J = \frac{\partial(u,v)}{\partial(x,y)} = 3$ then $J^* = \frac{\partial(x,y)}{\partial(u,v)} =$ ____ (A) 3 (B) 1 (C) $1/3$ (D) None of these	
	7.	$\int_0^{\pi/4}\int_0^1 r\, dr d\theta$ = ______ (A) $\pi/4$ (B) $\pi/8$ (C) $\pi/2$ (D) π	
	(b)		**07**
	1.	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots,\forall x$ is series expansion of the function____	

 (A) $\sin x$ (B) $\tan x$ (C) e^x (D) $\cos x$

2. The minimum value of $f(x, y) = x^4 + y^4 + 1$ is ____
   (A) 3 (B) 0 (C) 1 (D) 16

3. The series $\sum_{n=1}^{\infty} \frac{3n}{5n+1}$ is ________
   (A) Convergent (B) Divergent (C) Oscillating (D) can't decide

4. If in the equation of a curve, y occurs only as an even power then the curve is symmetrical about
   (A) $x - axis$ (B) $y - axis$ (C) $origin$ (D) None of these

5. A point (a, b) is said to be an extreme point if at (a, b)
   (A) $rt - s^2 > 0$ (B) $rt - s^2 < 0$ (C) $rt - s^2 = 0$ (D) $rt - s^2 \leq 0$

6. The value of $\lim_{x \to \infty} x^{1/x}$ is ______
   (A) ∞ (B) 1 (C) 0 (D) None of these

7. The asymptote to the curve $xy^2 = 4a^2(2a - x), a > 0$ is the line _____
   (A) $y = 0$ (B) $x = 2a$ (C) $x = 8a^3$ (D) $x = 0$

Q.2 **(a)** Expand $3x^3 + 8x^2 + x - 2$ in powers of $x - 3$. **03**

(b) Evaluate: **04**

(1) $\lim_{x \to 0} (\operatorname{cosec} x - \cot x)$ (2) $\lim_{x \to 1} (2 - x)^{\tan \frac{\pi x}{2}}$

(c) Expand $\tan^{-1}(x)$ up to the first four terms by Maclaurin's series and hence prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\right)$ **07**

Q.3

(a) Show that the function $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ has no limit as (x, y) approaches to $(0,0)$ **03**

(b) If $u = f(x^2 + 2yz, y^2 + 2zx)$ then prove that **04**

$$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0$$

(c) (1) State Euler's theorem on homogenous function of two variables. **04**

If $u = x^3 y^2 \sin^{-1}\left(\frac{y}{x}\right)$, show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 5u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 20u$

03

(2) If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, such that $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30,\ R_2 = 45$ and $R_3 = 90$ ohms.

Q.4 **(a)** Expand $e^x \cos y$ in powers of x and y up to terms of third degree. **03**

(b) Find the equation of the tangent plane and the normal line of the surface $x^2 + 2y^2 + 3z^2 - 12 = 0$ at the point $(1,2,-1)$. **04**

(c) Find the shortest and longest distance from the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$ **07**

Q.5

(a) Evaluate $\int_{x=0}^{1} \int_{y=0}^{x} \int_{z=0}^{\sqrt{x+y}} z \, dz \, dy \, dx$ **03**

(b) Evaluate $\iint_R xy \, dA$, where R is the region bounded by $x - axis$, ordinate $x = 2a$ and the curve $x^2 = 4ay$ **04**

(c) Change into polar coordinates and evaluate **07**

$$\int_0^a \int_0^{\sqrt{a^2 - y^2}} y^2 \sqrt{x^2 + y^2} \, dy \, dx$$

Q.6

(a) Discuss the convergence of the integral $\int_1^\infty e^{-x^2} dx$. **03**

(b) Check for the convergence of the following: **04**

(1) $\sum_{n=1}^{\infty} \frac{1}{1^2 + 2^2 + \ldots + n^2}$ (2) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$

(c)

(1) Check for convergence of the series $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \ldots$ **03**

(2) Find the radius of convergence and interval of convergence of the series **04**

$$1 - \frac{1}{2}(x-2) + \frac{1}{2^2}(x-2)^2 + \ldots + \left(-\frac{1}{2}\right)^n (x-2)^n + \ldots$$

Q.7 **(a)** The region between the curve $y = \sqrt{x}$ $0 \le x \le 4$, and the $x - axis$ is revolved about the $x - axis$ to generate a solid. Find its volume. **03**

04

(b) Expand $\sin\left(\pi/4 + x\right)$ in powers of x and hence find the approximate value of $\sin 44^0$.

(c) Trace the curve $xy^2 = a^2(a-x), a>0$ **07**

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