Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2017

Subject Code: 110008 **Date: 03/01/2018**

Subject Name: Maths - I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Attempt the following

(i) Prove that $f(x)=\begin{cases} x, x<0 \\ x^2, x\geq 0 \end{cases}$ is continuous at $x=0$. **3**

(ii) Find *c* of the mean value theorem for $f(x)=\log x; x\in[1,e]$. **4**

(b) Attempt the following

(i) Use Taylor's series to find the expansion of $\log_e x$ in powers of $(x-1)$. **3**

(ii) Evaluate $\lim_{x\to 0}\frac{\tan^2 x - x^2}{x^2\tan^2 x}$. **4**

Q.2 **(a)** Attempt the following

(i) Evaluate $\lim_{x\to \pi/2} \cos x \log \tan x$. **3**

(ii) Find the local extreme values of $f(x)=x^3-9x^2+15x+11$. **4**

(b) Attempt the following

(i) Test the convergence of $\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\ldots\infty$ **3**

(ii) Test the convergence of $\sum_{n=1}^{\infty}\frac{3^n n!}{n^n}$ **4**

Q.3 **(a)** Attempt the following

(i) Find $F'(x)$ for $F(x)=\int_3^{\sin x}\frac{1}{1+t^2}dt$. **3**

(ii) Check the convergence of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ **4**

(b) Find the area of the region between the *x*-axis and the graph of $f(x)=x^3-x^2-2x, -1\leq x\leq 2$. **07**

Q.4 **(a)** If $u=x^2y+y^2z+z^2x$, prove that $(i)\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=6(x+y+z)$ **07**

$(ii)\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=2(x+y+z)$

(b) If $u=f(2x-3y,3y-4z,4z-2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$ **07**

Q.5 **(a)** Attempt the following

(i) Evaluate $\iint_R e^{2x+3y} dA$ where R is the triangle bounded by *x=0, y=0* and *x+y=1*. **3**

(ii) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx$. **4**

(b) Evaluate $\int_0^{2a} \int_{x^2/4a}^{3a-x} (x^2 + y^2) dA$ by changing the order of integration. **07**

Q.6 **(a)** A vector field is given by $\overline{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$. Show that $\overline{F}$ is irrotational and find its scalar potential. **07**

(b) Verify Gauss' divergence theorem for $\overline{F} = yi + xj + z^2k$ for the cylindrical region S given by $x^2 + y^2 = a^2; z = 0$ and *z=h*. **07**

Q.7 **(a)** Attempt the following

(i) Using Green's theorem evaluate $\oint_C [(xy - x^2)dx + x^2ydy]$ along the closed curve C formed by *y=0, x=1* and *y=x*. **3**

(ii) Find the extreme values for $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. **4**

(b) Attempt the following

(i) Expand $e^x \log(1+y)$ in powers of *x* and *y*. **3**

(ii) Find the directional derivative of the function $\phi = x^2z + 2xy^2 + yz^2$ at the point (1,2,-1) in the direction of the vector $\overline{a} = 2i + 3j - 4k$. **4**

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