Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE –SEMESTER 1&2(NEW SYLLABUS)EXAMINATION- WINTER 2018

Subject Code: 3110014 **Date: 07-01-2019**

Subject Name: Mathematics - I

Time: 10:30 am to 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	State Cayley– Hamilton theorem. Find eigen values of A and A^{-1}, where $A=\begin{bmatrix}1 & 2\\ 0 & 4\end{bmatrix}$	**03**
	(b)	State L' Hospital's Rule. Use it to evaluate $\lim\limits_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{\sin^2 x}\right]$	**04**
	(c)	Investigate convergence of the following integrals: (i) $\int_5^\infty \frac{5x}{\left(1+x^2\right)^3}dx$ (ii) $\int_0^\infty \frac{x^{10}\left(1+x^5\right)}{\left(1+x\right)^{27}}dx$	**07**
Q.2	**(a)**	Test the convergence of series $\sum\limits_{n=1}^{\infty}\frac{4^n+5^n}{6^n}$	**03**
	(b)	State the p-series test. Discuss the convergence of the series $\sum\limits_{n=2}^{\infty}\frac{n+1}{n^3-3n+2}$	**04**
	(c)	State D'Alembert's ratio test and Cauchy's root test. Discuss the convergence of the following series: (i) $\sum\limits_{n=1}^{\infty}\frac{4^n\left(n+1\right)!}{n^{n+1}}$ (ii) $\sum\limits_{n=2}^{\infty}\frac{n}{\left(\log n\right)^n}$	**07**
		OR	
	(c)	Test the convergence of the series $\frac{1}{1\cdot 2\cdot 3}+\frac{x}{4\cdot 5\cdot 6}+\frac{x^2}{7\cdot 8\cdot 9}+\frac{x^3}{10\cdot 11\cdot 12}+\cdots;$ $x\ge 0$	**07**
Q.3	**(a)**	Reduce matrix $A=\begin{bmatrix}1 & 5 & 3 & -2\\ 2 & 0 & 4 & 1\\ 4 & 8 & 9 & -1\end{bmatrix}$ to row echelon form and find its rank.	**03**
	(b)	Derive half range sine series of $f\left(x\right)=\pi-x,\ 0\le x\le\pi$	**04**
	(c)	Find the eigen values and corresponding eigen vectors for the matrix A where $A=\begin{bmatrix}1 & 2 & 2\\ 0 & 2 & 1\\ -1 & 2 & 2\end{bmatrix}$	**07**

OR

Q.3 **(a)** Expand $e^{x\sin(x)}$ in power of x up to the terms containing x^6. **03**

(b) Solve system of linear equation by Gauss Elimination method, if solution exists. **04**

$x + y + 2z = 9;\ 2x + 4y - 3z = 1;\ 3x + 6y - 5z = 0$

(c) Find Fourier series of $f(x) = \begin{cases} x, & -1 < x < 0 \\ 2, & 0 < x < 1 \end{cases}$ **07**

Q.4 **(a)** Discuss the continuity of the function f defined as **03**

$$f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}; & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

(b) Define gradient of a function. Use it to find directional derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P(1,1,0)$ in the direction of $\bar{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. **04**

(c) Find the shortest and largest distance from the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$. **07**

OR

Q.4 **(a)** Find the extreme values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **03**

(b) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2)\,dydx$ by changing into polar coordinates. **04**

(c) (i) If $u = x^2y + y^2z + z^2x$ then find out $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z}$ **07**

(ii) If $x^3 + y^3 = 6xy$ then find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$

Q.5 **(a)** Evaluate $\iint_R y\sin(xy)\,dA$, where R is the region bounded by $x = 1$, $x = 2$, $y = 0$ and $y = \dfrac{\pi}{2}$. **03**

(b) By changing the order of integration, evaluate $\int_0^3 \int_y^3 \dfrac{xdxdy}{x^2 + y^2}$ **04**

(c) Find the volume below the surface $z = x^2 + y^2$, above the plane $z = 0$, and inside the cylinder $x^2 + y^2 = 2y$. **07**

OR

Q.5 **(a)** Evaluate integral $\iint_R \dfrac{rdrd\theta}{\sqrt{a^2 + r^2}}$ over the region R which is one loop of $r^2 = a^2 \cos 2\theta$ **03**

(b) Evaluate the integral $\int_1^e \int_1^{\log y} \int_1^{e^x} (x^2 + y^2)\,dzdxdy$. **04**

(c) Find the volume of the solid obtained by rotating the region R enclosed by the curves $y = x$ and $y = x^2$ about the line $y = 2$. **07**