Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (OLD) EXAMINATION – SUMMER-2019

Subject Code: 110015 **Date: 01/06/2019**

Subject Name: Vector Calculus And Linear Algebra

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Solve the following system by Gauss-Jordan elimination. **05**

$$3x+2y-z=-15$$
$$5x+3y+2z=0$$
$$3x+y+3z=11$$
$$-6x-4y+2z=30$$

(ii) Verify Cauchy-Schwarz inequality for the vectors $(-3,1,0)$ and $(2,-1,3)$. **02**

(b) (i) Find the inverse of A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. **05**

(ii) For which value of k are $u=(k,k,1)$ and $v=(k,5,6)$ orthogonal? **02**

Q.2 **(a)** (i) Use Cramer's rule to solve the following system. **05**

$$x+2z=6$$
$$-3x+4y+6z=30$$
$$-x-2y+3z=8$$

(ii) Find the rank of the following matrix. **02**

$$A=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$$

(b) (i) Prove that R^n is a vector space with the standard operations defined for R^n. **05**

(ii) Determine whether the set of all matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subspace of M_{22} or not. **02**

Q.3 **(a)** (i) Let $v_1=(1,2,1)$, $v_2=(2,9,0)$ and $v_3=(3,3,4)$. Show that the set $S=\{v_1,v_2,v_3\}$ is a basis for R^3. **05**

(ii) Determine whether the vectors $v_1=(-1,1,1)$, $v_2=(2,5,0)$ and $v_3=(0,0,0)$ of R^3 are linearly independent or linearly dependent . **02**

(b) (i)Let R^3 have the Euclidean inner product. Transform the basis $\{(1,1,1),(0,1,1),(0,0,1)\}$ into an orthogonal basis using gram-Schmidt process. **05**

(ii) Find the eigenvalues of A and A^2 where $A=\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$. **02**

Q.4 **(a)** Determine the algebraic and geometric multiplicity of **07**

$$A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$$

(b) (i) Let $u=(u_1,u_2)$ and $v=(v_1,v_2)$ be vectors in R^2. Verify that the weighted Euclidean inner product $<u,v>=3u_1v_1+2u_2v_2$ satisfies the four inner product axioms. **04**

(ii) Let R^4 have the Euclidean inner product. Find the cosine of the angle θ between the vectors $u=(4,3,1,-2)$ and $v=(-2,1,2,3)$. **03**

Q.5 **(a)** (i) Consider the basis $S=\{v_1,v_2\}$ for R^2, where $v_1=(-2,1)$ and $v_2=(1,3)$ and let $T:R^2\to R^3$ be the linear transformation such that $T(v_1)=(-1,2,0)$ and $T(v_2)=(0,-3,5)$. Find the formula for $T(x_1,x_2)$. Using it, find $T(2,-3)$. **07**

(b) Let $T_A:R^6\to R^4$ be multiplication by $A=\begin{bmatrix}-1 & 2 & 0 & 4 & 5 & -3\\ 3 & -7 & 2 & 0 & 1 & 4\\ 2 & -5 & 2 & 4 & 6 & 1\\ 4 & -9 & 2 & -4 & -4 & 7\end{bmatrix}$. **07**

Find the rank and nullity of T_A.

Q.6 **(a)** (i) Find the directional derivative of $f(x,y,z)=2x^2+3y^2+z^2$ at the point $P(2,1,3)$ in the direction the vector $\bar{a}=\hat{i}-2\hat{k}$. **03**

(ii)Obtain the reduced row echelon form of the matrix **04**

$$A=\begin{bmatrix}1 & -1 & 2 & -1 & -1\\ 2 & 1 & -2 & -2 & -2\\ -1 & 2 & -4 & 1 & 1\\ 3 & 0 & 0 & -3 & -3\end{bmatrix}$$

(b) (i) Find the gradient of $f(x,y,z)=2z^3-3(x^2+y^2)z+\tan^{-1}(xz)$ at $(1,1,1)$. **04**

(ii)Find the $curl\,\bar{F}$ at the point $(2,0,3)$ where $\bar{F}=ze^{2xy}\,\hat{i}+2xy\cos y\,\hat{j}+(x+2y)\hat{k}$. **03**

Q.7 **(a)** (i) Prove that $\bar{F}=(y^2\cos x+z^3)\hat{i}+(2y\sin x-4)\hat{j}+3xz^2\hat{k}$ is irrotational and find its scalar potential. **05**

(ii) State Divergence theorem. **02**

(b) State Green's theorem and using it, evaluate **07**

$$\oint_C (3x^2-8y^2)dx+(4y-6xy)dy$$

where C is the boundary of the region bounded by $y^2=x$ and $y=x^2$.

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