Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2018

Subject Code:110014 **Date: 21-05-2018**

Subject Name: Calculus

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a) (1)**	Evaluate $\lim_{x \to 1} \left(\frac{1}{\log x} - \frac{x}{x-1} \right)$	**03**
	(2)	Show that the sequence $\left\{ \frac{n}{n^2+1} \right\}$ is monotonic decreasing and bounded. Is it convergent?	**04**
	(b)	Find expansion of $\tan\left(x + \frac{\pi}{4}\right)$ is ascending powers of x upto terms in x^4 and find approximately the value of $\tan(43^\circ)$	**07**
Q.2	**(a)(1)**	Find expansion of $\log(1+x)$.	**03**
	(2)	Test the convergence of the series $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \ldots$	**04**
	(b)	Determine absolute or conditional convergence of the series. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3+1}$	**07**
Q.3	**(a) (1)**	Evaluate $\int_1^{\infty} \frac{1}{x^2} dx$	**03**
	(2)	Find the linearization of $f(x,y,z) = xy + yz + xz$ at the point (1,0,0)	**04**
	(b)	Trace the curve $r = a(1 - \cos\theta), a > 0$	**07**
Q.4	**(a)(1)**	Show that $f(x,y) = x^2 + 2y$ is continuous at (1,2).	**03**
	(2)	If $u = \tan^{-1}\left(\frac{x}{y}\right)$ *where* $x^2 + y^2 = a^2$ find $\frac{du}{dx}$.	**04**
	(b)	State Euler's theorem. If $u = tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$,prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial z^2} = -2sin^3 ucosu$	**07**
Q.5	**(a) (1)**	If $u = x^2 y^3, x = \log t, y = e^t$ find $\frac{du}{dt}$	**03**
	(2)	Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1).	**04**
	(b)	Change the order of integration and Evaluate for $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \; dydx$	**07**

Q.6 **(a)** **03**

(1) Evaluate $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1}\left(xz - y^{3}\right)dzdydx$

(2) State fundamental theorem of calculus. Use first fundamental theorem of calculus to find area under the curve f(x) given as an integrand. **04**

$\int_{1}^{2}\log x\,dx$

(b) Find the extreme values of the function $x^{3} + 3xy^{2} - 3x^{2} - 3y^{2} + 7$ **07**

Q.7 **(a)** **03**

(1) Evaluate $\int_{1}^{2}\int_{0}^{1}\left(1 + 3xy\right) dxdy$

(2) If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial(x, y)}{\partial(r,\theta)}$ *and* $\frac{\partial(r,\theta)}{\partial(x, y)}$ **04**

(b) Find the volume generated by revolving the area bounded by $2y = x^{2}, x = 4, y = 0$ about x -axis. **07**

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