Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd EXAMINATION (NEW SYLLABUS) – SUMMER 2018

Subject Code: 2110014 **Date: 21-05-2018**

Subject Name: Calculus

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) **07**

1. The sequence $\left\{\frac{\cos 2n}{n}\right\}$ converges to

(a) 1 (b) 0 (c) 2 (d) -1

2. Sum of the series $\sum_{n=0}^{\infty}\frac{4}{2^n}$ is

(a) 4 (b) 2 (c) 8 (d) 16

3. The coefficient of x^4 in the expansion of $\cos x$ is

(a) $\frac{1}{4!}$ (b) $-\frac{1}{4!}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

4. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x =$ ____________

(a) 0 (b) 1 (d) e (d) ∞

5. The curve $x^3+y^3=3xy$ is symmetric about

(a) x-axis (b) y-axis (c) line y = x (d) origin

6. Asymptote parallel to x-axis of the curve $3x^3+xy^2+xy=0$ is

(a) y=3 (b) y=1 (c) y=0 (d) not possible

7. The curve $r^2=a^2cos2\theta$ is not symmetric about

(a) initial line (b) line $\theta=\frac{\pi}{4}$ (c) line $\theta=\frac{\pi}{2}$ (d) pole

(b) **07**

1. If $u=ytan^{-1}(x/y)+xcot^{-1}(y/x)$, then xu_x+yu_y=_________

(a) 0 (b) u (c) 2u (d) 3u

2. For an implicit function $f(x,y)=c,$ the value of $\frac{dy}{dx}$ is

(a) $\frac{f_x}{f_y}$ (b) $\frac{f_y}{f_x}$ (c) $-\frac{f_x}{f_y}$ (d) $-\frac{f_y}{f_x}$

3. $\lim_{(x,y)\to(0,1)}\frac{y^2tan^{-1}x}{x}$ =________________

(a) 0 (b) 1 (c) -1 (d) ∞

4. $\int_0^1\int_0^y e^{x/y}\,dxdy$ =__________

(a) $\frac{e-1}{2}$ (b) $e-1$ (c) e (d) $e+1$

5. If $u=x-y$ and $v=x+y$, then the value of $J=\frac{\partial(x,y)}{\partial(u,v)}$ is ______

(a) 1 (b) -1 (c) $1/2$ (d) 1/4

6. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent when

(a) $p = 1$ (b) $p < 1$ (c) $p > 1$ (d) $p = 0$

7. The series $\sum_{n=1}^{\infty} \frac{n-1}{n+1}$ is__________

(a) divergent
(b) convergent and sum 1
(c) convergent and sum 2
(d) none of these

Q.2 **(a)** Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{n^3-3n+2}$ **03**

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{4^n+1}{5^n}$ **04**

(c) Evaluate (i) $\lim\limits_{x\to 0} \frac{2x-x\cos x-\sin x}{2x^3}$; (ii) $\lim\limits_{x\to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$. **07**

Q.3 **(a)** If $u = \log(x^2 + y^2)$, verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **03**

(b) If $u = r^m$, prove that $u_{xx} + u_{yy} + u_{zz} = m(m+1)r^{m-2}$, where $r^2 = x^2 + y^2 + z^2$. **04**

(c) Find the maxima and minima of the function $f(x,y) = x^2y - xy^2 + 4xy - 4x^2 - 4y^2$. **07**

Q.4 **(a)** If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. **03**

(b) Evaluate $\iint (6x^2 + 2y)\,dxdy$ over the region R bounded between $y = x^2$ and $y = 4$. **04**

(c) Change the order of integration and evaluate $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dydx$. **07**

Q.5 **(a)** If $z = xy^2 + x^2y,\ x = at^2,\ y = 2at$, find $\frac{dz}{dt}$. **03**

(b) If $u = e^{x^2+y^2-xy}$, then prove that **04**

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \ln u$;

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u \ln u(2\ln u + 1)$.

(c) **07**

(i) Check the absolute and conditional convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$.

(ii) Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-5)^n x^n}{n!}$.

Q.6 **(a)** Evaluate $\iint x dA$, over the region R bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **03**

(b) Expand $\cos\left(\frac{\pi}{4} + x\right)$ in powers of *x* by Taylor series. Hence find the value of $\cos 46°$. **04**

(c) Evaluate $\int_0^4 \int_{y/2}^{(y/2)+1} \frac{2x-y}{2}\,dxdy$ by applying the transformations $u = \frac{2x-y}{2}, v = \frac{y}{2}$. **07**

Q.7 **(a)** Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$. **03**

(b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2, x = 0$ about the line $y = 2$. **04**

(c) Trace the curve $y^2(a - x) = x^3,\ a > 0$. **07**

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