Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE-I - SEMESTER– 1st - EXAMINATION – SUMMER 2018

Subject Code: 110008 **Date: 21-05-2018**

Subject Name: MATHS-I

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Find the value of k so that the function given below is continuous at $x = 0$ **03**

$$f(x) = \begin{cases} \dfrac{\sin 3x}{2x} & x \neq 0 \\ k & x = 0 \end{cases}$$

(2) State Sandwich theorem on limit of sequences and using it find **04**

$\lim_{x \to 0} g(x)$, if $3 - x^3 \leq g(x) \leq 3\sec x$, $\forall x \in R$

(b) If $u = f(r)$, where $r^2 = x^2 + y^2$, prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{1}{r} f'(r)$ **07**

Q.2 **(a)** (1) Evaluate $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$ **03**

(2) Find the area of the region between the x-axis and the graph of **04**

$f(x) = x^3 - x^2 - 2x, \quad -1 \leq x \leq 2$

(b) State (1) Rolle's Theorem **07**

(2) The Mean Value Theorem

Find the value of c using Mean Value Theorem , for the function

$f(x) = 1 - x^2$, in $0 \leq x \leq 2$

Q.3 **(a)** (1) Find the gradient of $f(x, y, z) = 2z^3 - 3(x^2 + y^2)$ at the point (1,1,1). **03**

(2) Change the order of integration in the integral **04**

$\int_0^\infty \int_x^\infty \dfrac{e^{-y}}{y} \, dy \, dx$ and evaluate it.

(b) Trace the curve $r = a(1 + \cos\theta)$; $a > 0$ **07**

Q.4 **(a)** (1) Find the curl of $\vec{F} = (x^2 - z)\hat{i} + xe^z \hat{j} + xy\hat{k}$ **03**

(2) If $u = \left((x^2 + y^2 + z^2)\right)^m$ then find $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}$. **04**

(b) Find the local extreme values of the function **07**

$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$

Q.5 **(a)** (1) A fluid motion is given by **03**

$$\vec{v} = (y \sin z - \sin x)\,\hat{i} + (x \sin z + 2yz)\,\hat{j} + (xy \cos z + y^2)\,\hat{k}.$$

Show that the motion is irrotational.

(2) Expand $\sin\left(\frac{\pi}{4} + x\right)$ in powers of x. Hence find the value of $\sin 46^\circ$. **04**

(b) Find the point on the plane $x + 2y + 3z = 13$ closest to the point (1,1,1). **07**

Q.6 **(a)** State modified Euler's Theorem. Show that, if $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ **07**

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = \frac{1}{4}\sin 4u - \frac{1}{2}\sin 2u$$

(b) State Green's Theorem. Using Green's theorem, evaluate **07**

$\oint_C e^{-x}(\sin y\,dx + \cos y\,dy)$ where C is the rectangle with vertices $(0,0),\ (\pi,0),\ (\pi, \pi/2),\ (0, \pi/2)$.

Q.7 **(a)** Write the statement of Cauchy's integral test. Test the convergence of the series **07**

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^a}, \text{ for } 0 \le a \le 1.$$

(b) Find the volume of the solid of revolution of the area bounded by the curve $y = xe^x$ and the straight lines $x = 1,\ y = 0$ **07**

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