

**GUJARAT TECHNOLOGICAL UNIVERSITY**BE- IV<sup>th</sup> SEMESTER-EXAMINATION – MAY/JUNE- 2012**Subject code: 140001****Date: 18/05/2012****Subject Name: Mathematics-IV****Time: 10:30 am – 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a)** (1) Is  $\operatorname{Arg}(z_1 z_2) = \operatorname{Arg}(z_1) + \operatorname{Arg}(z_2)$ ? Justify. **02**

(2) Classify the poles of  $f(z) = \frac{1}{z^2 - z^6}$ . **02**

(3) Evaluate  $\int_C \frac{dz}{(z^2 + 1)^2}$  where  $C : |z + i| = 1$ . **03**

**(b)** (1) Let  $a > b > 0$ . Prove that **04**

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a^2 - b^2} \left( \frac{e^{-b}}{b} - \frac{e^{-a}}{a} \right).$$

(2) Define fixed point of bilinear transformation. Find fixed point of **02**

$$w = \frac{z+1}{z}$$

(3) Sketch  $S = \{z / -1 < \operatorname{Im}(z) < 2\}$ . Is it Connected? **01**

**Q.2 (a)** (1) Define Domain. Is the set  $|z - 1 + 2i| \leq 2$  domain? **02**

(2) Define  $\operatorname{Log}(x + iy)$ . Determine  $\operatorname{Log}(1-i)$ . **02**

(3) Discuss the rate of convergence of Newton Raphson's method. **03**

**(b)** (1) Prove  $\lim_{z \rightarrow 1} \frac{i\bar{z}}{3} = \frac{i}{3}$  by definition. **03**

(2) Verify that C – R equations are satisfied at  $z = 0$  for function **04**

$$f(z) = \begin{cases} \frac{-2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

**OR**

**(b)** (1) Suppose that  $f(z)$  and  $\overline{f(z)}$  both are analytic in a domain  $D$ . **04**  
Prove that  $f(z) = \text{constant}$ .

(2) Define Harmonic function. Show that **03**  
 $u = x \sin x \cosh y - y \cos x \sinh y$  is harmonic.

**Q.3 (a)** (1) Evaluate  $\int_C \bar{z} dz$  from  $z = 1 - i$  to  $z = 3 + 2i$  along the straight line. **05**

(2) Determine the bilinear transformation which mapping the points **02**  
 $0, \infty, i$  in to  $\infty, 1, 0$ .

**(b)** (1) Expand  $f(z) = \frac{1}{(z+2)(z+4)}$  valid for the region (i)  $|z| < 2$  **05**

(ii)  $2 < |z| < 4$  (iii)  $|z| > 4$

(2) Define the isolated singular point and give an example. **02**

**OR**

**Q.3 (a)** (1) Evaluate  $\int_C (x^2 - iy^2) dz$  along the parabola  $y = 2x^2$  from  $(1, 2)$  to **04**  $(2, 8)$ .

(2) Determine and sketch the image of  $|z| = 1$  under the transformation **03**  $w = z + i$ .

**(b)** (1) Find the series of  $f(z) = \frac{z}{(z-1)(z-4)}$  in terms of  $(z+3)$  valid for **05**  $|z+3| < 4$ .

(2) Find the residue of  $f(z) = \frac{1-e^z}{z^3}$  at  $z=0$ . **02**

**Q.4 (a)** (1) Find the negative root of  $x^3 - 7x + 3 = 0$  by bisection method up to **04** three decimal places.

(2) Describe Euler's Method for first order ordinary differential equation. **03**

**(b)** (1) Determine the largest eigen values of  $A = \begin{bmatrix} -1 & 1 & 4 \\ 10 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$  by power **04** method.

(2) Obtain the Newton – Raphson formula from Taylor's Theorem. **03**

**OR**

**Q.4 (a)** (1) Let  $f(40)=836$ ,  $f(50)=682$ ,  $f(60)=436$ ,  $f(70)=272$ . Use stirling's **04** formula to find  $f(55)$ .

(2) Find a root of  $x^4 - x^3 + 10x + 7 = 0$  correct to three decimal places **03** between  $a = -2$  and  $b = -1$  by Newton – Raphson Method.

**Q.4 (b)** (1) Derive Secant Method and solve  $xe^x - 1 = 0$  correct to three decimal **04** places between 0 and 1.

(2) Write  $f(x) = x^4 - 2x^3 + x^2 - 2x + 1$  in factorial notation and find **03**  $\Delta^4 f(x)$ .

**Q.5 (a)** (1) Determine  $y(0.1)$  and  $y(0.2)$  correct to four decimal places from **04**

$$\frac{dy}{dx} = 2x + y, \quad y(0) = 1.$$

Use fourth order Runge-Kutta method.

(2) Determine the polynomial by Newton's forward difference formula **03** from the following table.

$x$	0	1	2	3	4	5
$y$	-10	-8	-8	-4	10	40

**(b)** (1) Consider following tabular values. **03**

$x$	50	100	150	200	250
$y$	618	724	805	906	1032

Determine  $y(300)$ .

(2) Consider following tabular values. **04**

$x$	25.0	25.1	25.2	25.3	25.4	25.5	25.6
$y = f(x)$	3.205	3.217	3.232	3.245	3.256	3.268	3.280

Determine the area bounded by the given curve and X – axis between  $x=25$  to  $x=25.6$  by Trapezoidal rule and Weddle's rule.

**OR**

**Q.5 (a) (1)** Consider the following values.

**04**

<i>x</i>	10	11	12	13	14	15	16
<i>y</i>	1.02	0.94	0.89	0.79	0.71	0.62	0.55

Find  $\int_{10}^{16} y \, dx$  by Simpson's  $\frac{1}{3}$  rule and Weddle's rule.

**(2)** Solve  $\frac{dy}{dx} = 3 + 2x y$  where  $y(0) = 1$  for  $x = 0.1$  by Picard's method. **03**

**(b) (1)** Determine  $y(12)$  by Lagrange Interpolation from following values. **03**

<i>x</i>	11	13	14	18	20	23
<i>y</i>	25	47	68	82	102	124

**(2)** Solve the following system of linear equations by Gauss Jordan Method. **04**

$$2x + 5y - 3z = 1, \quad 5x + y + 4z = 2, \quad 7x + 3y + z = 4$$

\*\*\*\*\*