Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER-IV • EXAMINATION – SUMMER-2015

Subject Code: 140001 **Date: 26/05/2015**

Subject Name: Mathematics-IV

Time: 10.30AM-01.30PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** 1. From the following table estimate the number of students who obtain marks between 40-45. **05**

Marks	30-40	40-50	50-60	60-70	70-80
Number of students	31	42	51	35	31

2. Is $\operatorname{Arg}(z_1 z_2) = \operatorname{Arg}(z_1) + \operatorname{Arg}(z_2)$? Justify. **02**

(b) Show that $f(z) = \begin{cases} \sqrt{|xy|} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$ satisfy C-R equations at $z = 0$ but it is not differentiable at $z = 0$. **07**

Q.2 **(a)** Find the n[th] root of unity. Show that they are in Geometric progression and also show that their sum is zero. **07**

(b) Evaluate $\int_C \frac{z^4}{(z+1)(z-i)^2} dz$, where $C : 9x^2 + 4y^2 = 36$ by using Residue theorem. **07**

OR

(b) Find the Laurent series expansion of $f(z) = \frac{-1}{(z-1)(z-2)}$ for the region $(i)|z| < 1$, $(ii) 1 < |z| < 2$, $(iii)|z| > 2$ **07**

Q.3 **(a)** Evaluate $\int_0^\infty \frac{2x^2 - 1}{x^4 + 5x^2 + 4} dx$ **07**

(b) Evaluate $\oint_C \frac{z+4}{z^2 + 2z + 5} dz$, where C is $(i)|z| = 1$, $(ii)|z + 1 - i| = 2$, $(iii)|z + 1 + i| = 2$ **07**

OR

Q.3 **(a)** 1. If α and β are the roots of $x^2 - 2x + 2 = 0$, find the value of $\alpha^n + \beta^n$. Hence deduce that $\alpha^8 + \beta^8 = 32$. **04**

2. Define: Residue, Pole, Essential Singularity **03**

(b) 1. Prove that $\frac{\sin 7\theta}{\sin \theta} = 7 - 56 \sin^2 \theta + 112 \sin^4 \theta - 64 \sin^6 \theta$ and state the result which you have used. **04**

2. Define: Mobious transformation, Harmonic function, Analytic function **03**

Q.4 **(a)** By using Lagrange's formula find y when x=10 **07**

x	5	6	9	11
y	12	13	14	16

(b) Solve by using Gauss-Seidel method correct to 5 decimal places. **07**

$$8x - 3y + 2z = 20$$

$$6x + 3y + 12z = 35$$

$$4x + 11y + z = 33$$

OR

Q.4 **(a)** Find the root of $x \log_{10} x = 1.2$ by using Newton's Raphson method correct up to six decimal places. **07**

(b) Using Newton's divided difference formula find a polynomial and also find $f(-1)$ and $f(6)$. **07**

x	1	2	4	7
f(x)	10	15	67	430

Q.5 **(a)** Use fourth order RK method to find the value of y at x=1, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ with h=0.5. **07**

(b) Find a root of $x^3 - 5x + 3 = 0$ by Bisection method correct up to four decimal places. **07**

OR

Q.5 **(a)** Use Simson's $1/3$ rd rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking n=6. **07**

(b) Using Taylor's series method to solve $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, at $x = 0.2$, $h = 0.1$. **07**

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