Seat No.: $\qquad$

# GUJARAT TECHNOLOGICAL UNIVERSITY <br> MCA SEMESTER -I Regular/Remedial EXAMINATION 

Subject code: 2610003
Date: 05/01/2013
Subject Name: Discrete Mathematics for Computer Science
Time: 02:30 pm - 05:00 pm
Total Marks: 70
Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q-1 (a) Let $X=\{1,2,3,4\}$ and $R=\{(1,1),(1,4),(4,1),(4,4),(2,2),(2,3),(3,2),(3,3)\}$
(i) write properties of ${ }^{R}$.
(ii) write the matrix of ${ }^{R}$.
(iii) Sketch the graph of ${ }^{R}$.From the graph, write partition of ${ }^{X}$.
(iv) Is ${ }^{R}$ is a function ? justify your answer.
(b)
(i) Define a group. Show that $\left(Z_{6},+_{6}\right)$ is a group.
(ii) Define a subgroup of a group. Write all the subgroups of $\left(Z_{6},+_{6}\right)$.

What is the relation between order of a subgroup and order of a finite group?
(iii) Define a cyclic group. Is $\left(Z_{6},+_{6}\right)$ cyclic? Justify your answer.

Q-2 (a) (I) Give an illustration of
(i) A bounded lattice which is complemented but not distributive.
(ii) A bounded lattice which is distributive but not complemented.
(iii) A bounded lattice which is both distributive and complemented.
(II) Draw Hasse diagram of following Posets .
(i) $\left(S_{30}, D\right)$
(ii) $\left(S_{36}, D\right)$
(b) Define an equivalence relation. Let ${ }^{Z}$ be the set of integers and ${ }^{R}$ be the relation " congruence modulo ${ }^{5}$ " defined as $R=\{(x, y) / x \in Z \wedge y \in Z \wedge(x-y)$ is divisible by 5$\}$

Show that ${ }^{R}$ is an equivalence relation on ${ }^{Z}$. Determine the equivalence classes generated by the elements of ${ }^{Z}$.

## OR

(b) (I) Define a partial order relation. Let ${ }^{X}$ be the set of positive integers and ${ }^{R}$ be the relation " $x D y$ " where ${ }^{D}$ stands for "divides". Show that ${ }^{D}$ is a partial order relation on ${ }^{X}$.
(II) Define a Poset. In Poset ${ }^{\left(S_{75}, D\right)}$ find lower bounds of the subsets
$A=\{5,15,25\}$ and $B=\{3,15,75\}$.
Q-3 (a) (I) Let ${ }^{p}$ and ${ }^{q}$ be two statements. Define conditional statement ${ }^{p \rightarrow q}$.Write its converse, inverse and contrapositive.
(II) Define universal quantifier. Write the universal quantification of the sentence: $x^{2}+x$ is an even integer, where ${ }^{x}$ is an even integer.
Is this universal quantification a true statement?
(b) (I) Define an abelian group. Show that in a group ${ }^{(G, *)}$, if for any $a, b \in G$,
$(a * b)^{2}=a^{2} * b^{2}$ then $(G, *)$ must be abelian.
(II) Define: (i) A group homomorphism
(ii) Kernel of homomorphism

Show that kernel of homomorphism g is a subgroup of $(G, *)$, where g is a homomorphism from ${ }^{(G, *)}$ to $(H, \Delta)$.

## OR

Q-3 (a) (I) Define existential quantifier. Write existential quantification of the sentence :
${ }^{x}$ is a prime integer where x is an odd integer.
Is this existential quantification a true statement ?
(II) Test the validity of logical consequence :

All dogs fetch.
Ketty does not fetch.
Therefore Ketty is not a dog.
(III) Using truth table, prove that $\sim(p \rightarrow q) \equiv p \wedge(\sim q)$.
(b) (I) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian.
(II) Let ${ }^{\left(H_{1}, *\right)}$ and $\left(H_{2}, *\right)$ be subgroups of a group ${ }^{(G, *)}$. Show that $\left(H_{1} \cap H_{2}, *\right)$ is also a subgroup of $(G, *)$. Will ${ }^{\left(H_{1} \cup H_{2}, *\right)}$ be also a subgroup of $(G, *)$ ?
Q-4 (a) (I) Define a sublattice. Write any four sublattices of $\left(S_{12}, D\right)$.
(II) Define a Boolean Algebra. Show that in a Boolean Algebra

$$
(a * b)^{\prime}=a^{\prime} \oplus b^{\prime} \quad \text { and } \quad(a \oplus b)^{\prime}=a^{\prime} * b^{\prime}
$$

(b) (I) Define a symmetric Boolean expression. Determine whether following expressions are symmetric or not.
(i) $\left(x_{1} * x_{2}{ }^{\prime}\right) \oplus\left(x_{1}{ }^{\prime} * x_{2}\right)$
(ii) $\left(x_{1}{ }^{\prime} * x_{2}{ }^{\prime}\right) \oplus\left(x_{1} \oplus x_{2}\right)$
(II) Use the Quine Mc Clusky method to simplify the sum-of-product expression: $f(a, b, c, d)=\sum(10,12,13,14,15)$.

OR
Q-4 (a) Define direct product of two lattices. Show that the lattice $\left(S_{216}, D\right)$ is [7] isomorphic to the direct product of lattices $\left(S_{8}, D\right)$ and $\left(S_{27}, D\right)$.
(b) (I) Show that in a Boolean Algebra $a \leq b \Leftrightarrow b^{\prime} \leq a^{\prime}$.
(II) Use Karnaugh map representation to find a minimal sum-of-product [4] expression $f(a, b, c, d)=\sum(0,1,2,3,13,15)$.
Q-5 (a) (I) Show that the sum of indegrees of all the nodes of a simple digraph is equal to the sum of outdegrees of all its nodes and that this sum is equal to the number of edges of the graph.
(II) Define isomorphic graphs. State whether the following digraphs are [4] isomorphic or not. Justify your answer.

(b) (I) Define weakly connected, unilaterally connected and strongly connected [3] graphs.
(II) Define weak, unilateral and strong components. Find the strong, unilateral and weak components for the following digraph.


Q-5 (a) Define node base of a simple digraph. Find the reachability set of all nodes for the following digraph.

(b) (I) Define (i) A directed tree (ii) A Binary tree (iii) A complete ${ }^{m}$-ary tree.
(II) Show that in a complete binary tree the total number of edges is given by $2\left(n_{i}-1\right)$ where ${ }^{n_{i}}$ is the number of terminal nodes.

