Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–I • EXAMINATION – WINTER 2013

Subject Code: 2610003 Date: 23-12-2013

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 02:30 pm TO 05:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Let $X=\{1,2,3,4,5\}$ and R,S,T be relations on X as follows :

$R=\{(x,y)/x+y=5\}$ $S=\{(1,2),(3,4),(2,2)\}$ $T=\{(4,2),(2,5),(3,1),(1,3)\}$

(i) Write properties of R. **03**

(ii) Write matrix of R. **01**

(iii) Find $S\circ T, R\circ S$ *and* $S\circ R$. **03**

(b) (i) Define a group. Show that $(Z_6,+_6)$ is a group. Is it cyclic? **03**

(ii) Let $(G,\ *)$ be a group. Let $|G|=5$. How many subgroups are there of G? Why ? **01**

(iii)Write (a) degree and (b) order of permutation group of $(S_3\ ,\Diamond)$.Is this group abelian? Justify your answer. **03**

Q.2 (a) (i) Define a partial order relation. Let X be the set of positive integers and R be the relation "xDy" where D stands for "divides". Show that D is a partial order relation on X . **04**

(ii) Define an equivalence relation. Given a set $S=\{1,2,3,4,5,6\}$. Find the equivalence relation on S which generates the partition $\{\overline{1,2}\ ,\overline{3,4},\overline{5,6}\}$. Draw the graph of the relation. **03**

(b) (i) Draw Hasse Diagram of the following posets. **04**

(a) $(S_{45}\ ,D)$ (b) $(S_{32}\ ,D)$.

(ii) Show that the relation "congruence modulo m" given by $\equiv\ =\{(x,y)/(x-y)\ \textit{is divisible by}\ m\}$ over the set of positive integers is an equivalence relation. **03**

OR

(b) (i) Give an example of a set X such that $(\rho(X)\ ,\subseteq)$ is a totally ordered set. **02**

(ii) Give a relation which is both, a partial order relation and an equivalence relation on a set. **02**

(iii) Write a short note on applications of relations to database theory. **03**

Q.3 (a) (i) Let P and Q be two statements. Write (a) converse and (b) contra positive of implication $P\rightarrow Q$. **02**

(ii) What is the truth value of the quantification $\exists xP(x)$? The domain of the discourse is the set of all real numbers. **02**

(a) $P(x): x+1=1$ (b) $P(x):(x+1)^2<0$.

(iii) Test the validity of the logical consequence : **03**

(1) All drivers take a driving test. (2) Tom did not take the driving test.

(3) Therefore Tom is not a driver.

(b) (i) Define an abelian group. Show that in a group $(G, *)$, if for any $a, b \in G$, $(a*b)^2 = a^2 * b^2$ then $(G, *)$ must be abelian. **03**

(ii) Define left coset of a subgroup H in a group G, determined by the element $a \in G$. Find the left cosets of $\{[0],[3]\}$ in the group $(Z_6, +_6)$. **02**

(iii) Define kernel of a group homomorphism g from a group $(G, *)$ to (H, Δ). Show that it is never empty. **02**

OR

Q.3 (a) (i) Determine the truth value of each of the following statement. **02**

(a) Either today is Monday or $\sqrt{7}$ is a real number.
(b) If Mickey is in Florida, then 17 is an odd integer.

(ii) What is the truth value of the quantification $\forall x P(x)$? The domain of the discourse is the set of all positive integers. **02**

(a) $P(x):(x+1)(x+2)$ is an even integer.
(b) $P(x):(x+1)>x$

(iii) Test the validity of the logical consequences. **03**

Everyone who studies logic, is good in reasoning.
Lata is good in reasoning.
Therefore, Lata studies logic.

(b) (i) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian. **03**

(ii) Show that $(\{[1],[4],[13],[16]\}, X_{17})$ is a subgroup of (Z^*_{17}, X_{17}). **02**

(iii) Show that every subgroup of a cyclic group is normal. **02**

Q.4 (a) (i) Define a sublattice. Show that every interval of a lattice is a sublattice. **02**

(ii) Show that in a lattice if $a \le b$ and $c \le d$, then $a*c \le b*d$ **02**

(iii) Define (a) A complemented lattice. **03**
(b) A distributive lattice.
Give one illustration of a lattice, which is complemented and distributive both.

(b) (i) In any Boolean Algebra, show that $a = b \Leftrightarrow ab' + a'b = 0$ **03**

(ii) Use Karnough map representation to find a minimal sum-of-products expression $f(a,b,c,d) = \sum(5,7,10,13,15)$ **04**

OR

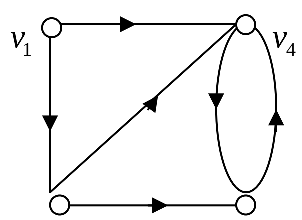
Q.4 (a) (i) Define direct product of two lattices. Show that the lattice (S_{36}, D) is isomorphic to the direct product of lattices (S_4, D) and (S_9, D). **05**

(ii) Define a complemented lattice. Which of the two lattices (S_n, D) for $n = 42$ and $n = 45$ are complemented ? **02**

(b) (i) In any Boolean algebra, show that $a = 0 \Leftrightarrow ab' + a'b = b$. **03**

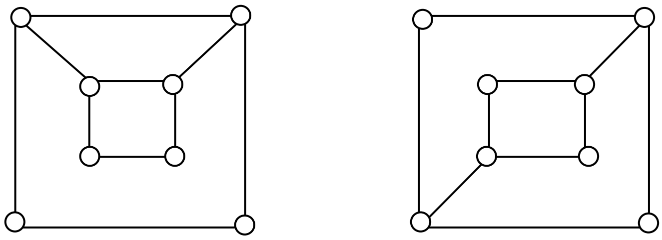
(ii)Use the Quine Mc Clusky method to simplify the sum-of-products expression $f(a,b,c,d) = \sum(10,12,13,14,15)$ **04**

Q.5 (a) (i) Give three different elementary paths from v_1 to v_3 for the digraph given in Figure -1. What is the shortest distance between v_1 and v_3 ? **03**



v_2 v_3 Figure-1

(ii) Define isomorphic graphs. State whether the following digraphs are isomorphic or not. Justify your answer. **04**



(b) (i) Define Reachable set of a node v. Find the reachable sets of $\{v_1\}, \{v_3\}$ and $\{v_5\}$ for the digraph given in Figure-2. **03**

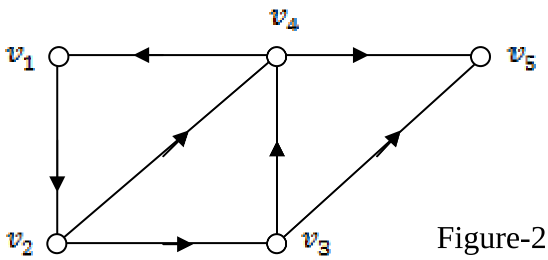


Figure-2

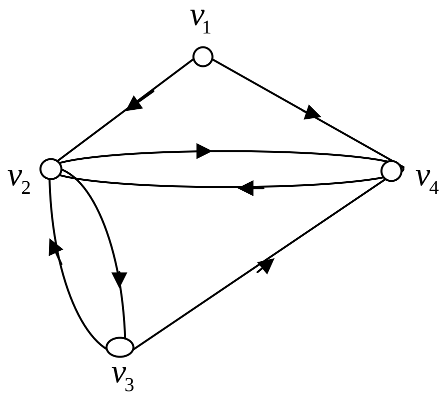
(ii) Define: (a) A Binary tree (b) A complete m-ary tree **02**

(iii) State whether digraphs given in Figure-1 and Figure-2 are strongly, weakly or unilaterally connected. **02**

OR

Q.5 (a) (i) Define node base of a simple digraph. Prove that in an acyclic simple digraph a node base consists of only these nodes whose indegrees is zero. **03**

(ii) Obtain the adjacency matrix A of the following digraph. Find the elementary paths of lengths 1 and 2 from v_1 to v_4 .Verify the results by calculating A^2. **04**



(b) (i) Define a complete binary tree. Show that in a complete binary tree the total number of edges is given by $2(n_i - 1)$, where n_i is the number of terminal nodes. **03**

(ii) Define Isomorphic graphs. What are the necessary conditions for two graphs to be isomorphic? Are they sufficient also? Justify your answer. **04**

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