Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA Sem-I Remedial Examination April 2010

Subject code: 610003

Subject Name: Discrete Mathematics For Computer Science

Date: 06 / 04 / 2010 **Time: 12.00 noon – 02.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Answer the following:

(i) Express the following using predicates, quantifiers, and logical connectives. Also verify the validity of the consequence. **04**

Everyone who graduates gets a job.
Ram is graduated.
Therefore, Ram got a job.

(ii) Prove by contradiction that $\sqrt{2}$ is an irrational number. **03**

(b) Draw Hasse Diagram of the poset $\langle\{2,3,5,6,9,15,24,45\},D\rangle$. Find **07**

(i) Maximal and Minimal elements
(ii) Greatest and Least members, if exist.
(iii) Upper bound of $\{9,15\}$ and l.u.b. of $\{9,15\}$, if exist.
(iv) Lower bound of $\{15,24\}$ and g.l.b. of $\{15,24\}$, if exist.

Q.2 **(a)** When a poset said to be lattice? Explain. Is every poset a lattice? Justify. Is the poset $\langle\{\varnothing,\{p\},\{q\},\{p,q,r\}\}, \subseteq\rangle$ lattice? **07**

(b) Show that the lattice $\langle S_n, D\rangle$ for $n = 100$ is isomorphic to the direct product of lattices for $n = 4$ and $n = 25$. **07**

OR

(b) With proper justification give an example of **07**

(1) A bounded lattice which is complemented but not distributive.
(2) A bounded lattice which is distributive but not complemented.
(3) A bounded lattice which is neither distributive nor complemented.
(4) A bounded lattice which is both distributive and complemented.

Q.3 **(a)** Answer the following:

(i) Define sub-Boolean algebra. State the necessary and sufficient condition for a subset becomes sub-algebra. Find all sub Boolean algebra of $\langle S_{110}, D\rangle$. **05**

(ii) Prove the following Boolean identities: **02**

(1) $(x'\oplus y)*(x\oplus y)=y$ (2) $(x\oplus y\oplus z)*(y\oplus z)=(y\oplus z)$

(b) Use the Quine-Mccluskey algorithm to find the prime implicants and also obtain a minimal expression for function: $f(a,b,c,d)=\sum(15,14,13,6,5,2,1)$. **07**

OR

Q.3 **(a)** Use Karnaugh map to find a minimal sum-of-product expression for the function given by $\sum(0,1,2,3,6,7,13,14)$ in four variables w, x, y, z. **07**

(b) Answer the following:

(i) Given an expression $\alpha(a,b,c,d)=\sum(2,3,6,8,12,15)$, determine the value of $\alpha(3,5,10,30)$ where $3,5,10,30 \in \langle S_{30}, D\rangle$. **04**

(ii) Find the sum of products expansions of Boolean functions $f(x,y,z)=(x+z)y$ **03**

Q.4 **(a)** Define group homomorphism; prove that group homomorphism preserves identities, inverses and subgroups. **07**

(b) Define cyclic group. Find generators of $\langle Z_{12}, +_{12}\rangle$. Also find its all subgroups. Which subgroups are isomorphic to $\langle Z_4, +_4\rangle$? Justify. **07**

OR

Q.4 **(a)** Show that if every element in a group is its own inverse, then the group must be abelian. Is the converse true? Justify. **07**

(b) Define symmetric group $\langle S_3, \Diamond\rangle$. Write its composition table. Determine all the proper subgroups of $\langle S_3, \Diamond\rangle$. Which subgroup is normal subgroup? Support your answer with reason. **07**

Q.5 **(a)** Define isomorphic graphs. Determine whether the digraphs G and H given in figure – 1 (a), (b) are isomorphic. **07**

(b) Define node base of a digraph. Find all node base of the digraph shown in figure – 2. List out all the properties of a node base. Explain why no node in node base is reachable from any other node in node base. **07**

OR

Q.5 **(a)** Define: path, simple path, elementary path. For the graph given in Figure – 3: **07**

(i) Find an elementary path of length 2 from v_1 to v_3.

(ii) Find a simple path from v_1 to v_3, which is not elementary.

(iii) Find all possible paths from node v_2 to v_4 and how many of them are simple and elementary?

(b) Define a directed tree. Draw the graph of the tree represented by $\left(A\left(B\left(C(D)(E)\right)\left(F(G)(H)(J)\right)\right)\left(K(L)(M)\left(N(P)\left(Q(R)\right)\right)\right)\right)$ Obtain the binary tree corresponding to it. **07**

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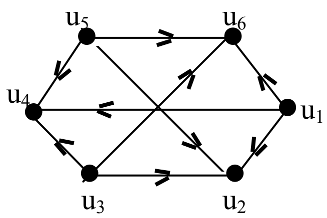
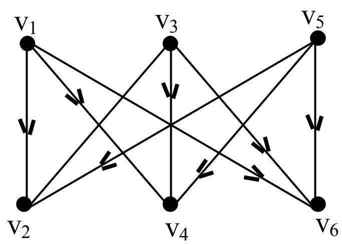


Graph G:
Figure – 1 (a)



Graph H:
Figure – 1 (b)

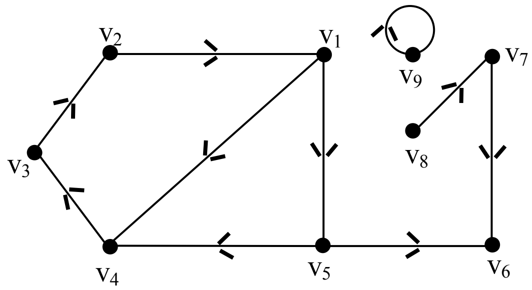


Figure – 2

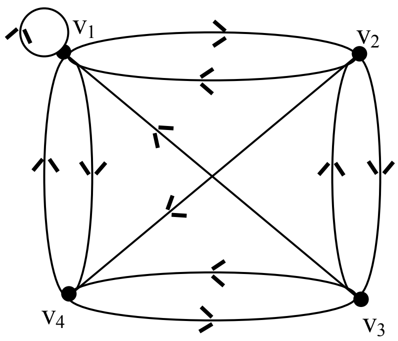


Figure – 3