$\qquad$
$\qquad$

## GUJARAT TECHNOLOGICAL UNIVERSITY <br> MCA - SEMESTER-I • EXAMINATION - SUMMER • 2015

## Subject Code: 610003

Date: 07-05-2015

## Subject Name: Discrete Mathematics for Computer Science

Time: 10:30 am-01:00 pm
Total Marks: 70

## Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 (a) (i)Test the validity of the following logical consequence.

If Chris studies, then he will pass the class test. If Chris does not play cards, then he will study. Chris did not pass in the class test. Therefore, Chris played cards.
(ii) Give direct proof to show that if x is an even integer then $\mathrm{x}^{2}+\mathrm{x}$ is also an even integer.
(b) Explain Universal quantification, Universal quantifier, Existential quantification and Existential quantifier.
Q. 2 (a) Draw Hasse diagram of following lattices
(i) $\left(\mathrm{S}_{36, \mathrm{D}} \mathrm{D}\right)$
(ii) $\left(S_{45}, D\right)$
(b) Let $(\mathrm{L}, \leq)$ be a lattice and $\mathrm{a}, \mathrm{b} \in \mathrm{L}$ then prove that
$\mathrm{a} \leq \mathrm{b} \Leftrightarrow \mathrm{a} * \mathrm{~b}=\mathrm{a} \Leftrightarrow \mathrm{a} \oplus \mathrm{b}=\mathrm{b}$
(b) State and prove absorption law for lattice < L , $\leq$ 》. 07
Q. 3 (a) Find a minimal sum-of-product using K-map
(i) $\alpha(x, y, z)=x y z+x y z '+x y^{\prime} y z '+x^{\prime} y^{\prime} z$
(ii) $\alpha(x, y, z)=x y z+x y z '+x y^{\prime} z+x^{\prime} y z+x^{\prime} y^{\prime} z$.
(b) Find all Sub Boolean algebras of Boolean algebra $\left\langle\mathrm{S}_{30}, \Lambda, \mathrm{~V},{ }^{`}, 0,1\right\rangle$. $\mathbf{0 7}$

## OR

Q. 3 (a) Define:
i) Join irreducible elements.
ii) Atoms of a Boolean algebra.

Determine Join-irreducible elements and atoms of ( $\mathrm{S}_{210}, \mathrm{D}$ )
(b) Let $X=\{a, b, c\}$ then show that $<P(X), \cap, \cup, \varnothing, X>$ is complemented lattice. 07
Q. 4 (a) Find all left and right cosets of $\mathrm{H}=\left\{\mathrm{p}_{1}, \mathrm{P}_{5}, \mathrm{p}_{6}\right\}$ in ( $\mathrm{S}_{3}, \mathrm{D}$ ). Where ( $\mathrm{S}_{3}, \mathrm{D}$ ) is a $\mathbf{0 7}$ symmetric group.
(b) Show that cyclic group is abelian group but converse is not true. 07 OR
Q. 4 (a) Define cyclic group and subgroup. Show that ( $\left(\mathrm{Z}^{*}{ }_{11}, \mathrm{X}_{11}\right)$ is cyclic group. Find its all generators. Also find its all subgroups.
(b) Define Group Isomorphism. Show that $\left(\mathrm{Z}_{6},+_{6}\right)$ is isomorphic to $\left(\mathrm{Z}_{7}{ }^{*}, \times_{7}\right)$ 07
Q. 5 (a) Obtain binary tree corresponding to following tree.
(b) Explain why following graphs are not isomorphic?

$v_{4}$

OR
Q. 5 (a) Define complete binary tree. Show through two examples with $n t=7$ and $n t=8$ of complete binary trees that the total number of edges is given by $2(\mathrm{nt}-1$ ), where $n t$ is the number of terminal nodes.
(b) Define: Node Base. Find all node bases for the following graph


