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## GUJARAT TECHNOLOGICAL UNIVERSITY <br> M. E. - SEMESTER - I • EXAMINATION - WINTER • 2014

Subject code: 2710002
Date: 06-01-2015
Subject Name: Computational Method for Mechanical Engineering
Time: 02:30 pm - 05:00 pm
Total Marks: 70
Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 (a) Find the steady state oscillation of the mass spring system governed by the equation

$$
y^{\prime \prime}+3 y^{\prime}+2 y=20 \cos 2 t
$$

(b) The model of "sealed container with atomic waste dumped into the ocean" is
$m \frac{d v}{d t}=W-B-k v, v(0)=0$ where W is the weight of the container, B the buoyancy force of the water and $-k v$ is the drag. Solve the equation to obtain $v(t)$ and integrate to get $y(t)$ such that $y(0)=0$
Q. 2 (a) Using Laplace Transform solve: $y^{\prime \prime}-y=t ; \quad y(0)=y^{\prime}(0)=1$.
(b) Find one root of $\mathrm{e}^{\mathrm{x}}-3 \mathrm{x}=0$, correct to two decimal places using the method of Bisection.

OR
(b) Find out what type of conic section the following quadratic form represents and transform it to principal axes: $17 \mathrm{x}_{1}{ }^{2}-30 \mathrm{x}_{1} \mathrm{x}_{2}+17 \mathrm{x}_{2}{ }^{2}=128$
Q. 3 (a)

Find a matrix $P$ that diagonalizes the matrix $A=\left[\begin{array}{ccc}-1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3\end{array}\right]$ and determine $\mathrm{P}^{-1} \mathrm{AP}$.
(b) Verify dimension theorem for the linear transformation $T: R^{4} \rightarrow R^{3}$ given by the formula

$$
\begin{gathered}
T(x, y, z, w)=(4 x+y-2 z-3 w, 2 x+y+z-4 w, 6 x-9 z+9 w) . \\
\text { OR }
\end{gathered}
$$

Q. 3 (a) Solve the following equation using Gauss-Jordan method.
$10 x+y+z=12$
$2 x+10 y+z=13$
$x+y+5 z=7$
(b) Compute the flux of water through $S$ : $|x| \leq 1,|y| \leq 3,|z| \leq 2$ if the velocity vector is $V=F=\left[x^{2}, 0, z^{2}\right]$ (As $\mathrm{F}=\rho V$ and density $\rho=1$ for water).
Q. 4 (a) Lines $L_{1}$ and $L_{2}$ are given by following parametric equations respectively.
$\mathrm{x}=1+6 \mathrm{t}, \quad \mathrm{y}=2-4 \mathrm{t}, \quad \mathrm{z}=-1+3 \mathrm{t} ;$ $x=4-3 p, \quad y=2 p, \quad z=-5+4 t$, where parameters $p$ and $t$ takes all real values. Find point of intersection and angle between two lines.
(b) A periodic motion observed on the oscilloscope is illustrated in figure 1.

Represent this motion by harmonic series (Fourier series).
(ases)
Figure 1

## OR

Q. 4 (a) The following table gives the marks secured by students.

| Range of marks | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 32 | 45 | 54 | 31 | 34 |

Find the number of students who got marks between 50 and 55.
(b) A body executes damped forced vibrations given by the equation.
$\frac{d^{2} x}{d t^{2}}+2 k \frac{d x}{d t}+b^{2} x=e^{-k t} \sin \omega t$
Solve the equation when $\omega^{2} \neq b^{2}-k^{2}$
Q. 5 (a) By method of least squares, find the curve $y=a x+b x^{2}$ that best fit the following data:

| X | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Y | 1.9 | 5.4 | 9.3 | 14.6 | 18.8 |

(b) 1. Using Poisson distribution, find the probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials.
2. If on an average one ship in every fifteen is wrecked, find the probability that out of seven ships expected to arrive, four at least will arrive safely.

## OR

Q. 5 (a) Determine the largest Eigen value and corresponding eigenvector of the matrix

$$
A=\left[\begin{array}{ccc}
1 & 1 & 0 \\
1 & 2 & 1 \\
-1 & 1 & 2
\end{array}\right]
$$

(b) A 20 kg mass is resting on a spring of $4700 \mathrm{~N} / \mathrm{m}$ and dashpot of $147 \mathrm{~N}-\mathrm{Sec} / \mathrm{m}$ in parallel. If a velocity of $0.10 \mathrm{~m} / \mathrm{sec}$ is applied to the mass at the rest position, what will be the displacement from the equilibrium position at the end of first second?

